Simulating Linear Matched *SAW* Filters Based on Delta Function

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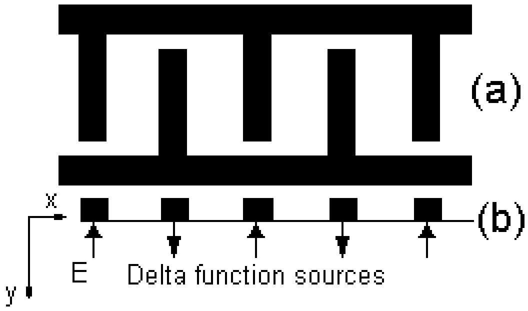
Abstract: In this paper, a mathematical model has been derived using the superposition principle and the delta function model; this model can be applied to simulate the frequency response of linear matched surface acoustic wave (SAW) filters. The simulation and the test results are similar in center frequency and 3-dB bandwidth, but the insertion loss is different, because of the time gating method used in testing.

1. Introduction

The delta function model provides basic information on the transfer function response of linear matched *SAW* filters. It can only yield a relative insertion loss as a function of frequency. When the voltage is applied at the electrodes of an *IDT* structure of Fig. 1.a, it produces an electric field in the gaps between electrodes. At any instant, adjacent electrodes have opposite voltage polarity and opposite charges that accumulate at the edges of the *IDT* fingers. The resultant charge distribution can be modeled as delta function sources of electric field intensity E_y at the finger edges, *see* Fig. 1.b. The summation of these delta sources can be used to simulate the resultant electric field intensity, and yield the frequency response *H (f)* (Campbell, 1998 and Panasik, 1992).



(a) Bi-directional *IDT* with uniform apodization.
(b) Delta-function modeling of *E*-field distribution.

Fig. 1: Bi-directional *IDT* of uniform apodization and its delta-function modeling.

2. Derivation of the Mathematical model

Fig. 2 represents a linear **matched** *SAW* filter with unapodized input and output *IDTs*, each group of *IDTs* forms a rung (Campbell, 1998 and Panasik, 1981), so, we have several input and output rungs. Now, let the first *IDT* of the first input rung be located at the zero *x-axis* and consider it as a reference for the other input and output rungs, as shown in Fig. 2.

To calculate the frequency response of the input and the output rungs, we will select a reference point to accumulate all the summations on it. For simplicity, we have selected the zero *x*-axis as a reference point. The general model for calculating the frequency response of any input or output rung will take the following form (see Eq. 1) (Issa, 2000).

$$H_{iq/ol}(f) = \sum_{n=1}^{N} (-1)^{n} \exp(-i\beta x_{n}) = [\text{F. R. Rung}]\ \ \text{iq/ ol} \tag{1}$$

Whereas,

$H_{iq/ol}(f)$: Is the frequency response of *q* input (iq) rung **<u>or</u>** *l* output (*ol*) rung = [F. R. Rung] $_{\text{iq/ ol}}$.

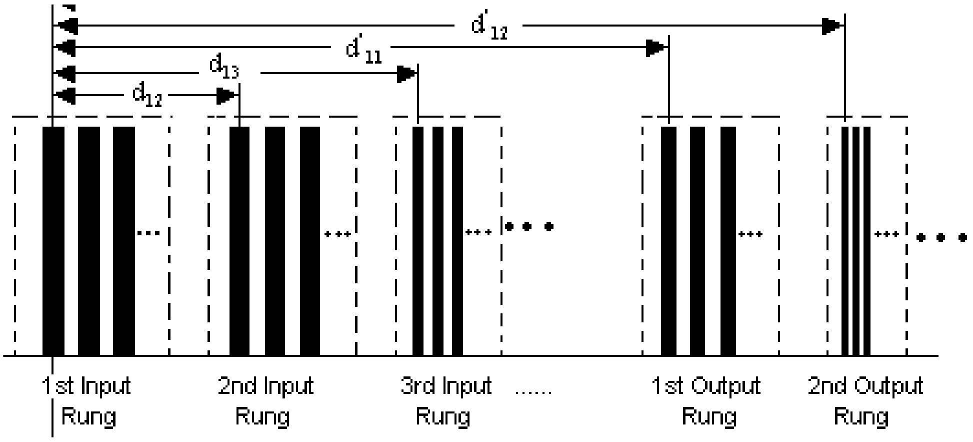
$(-1)^{n}$: Refers to the alternating electrode polarity.

A_n : Is the amplitude that is proportional to the finger apodization overlap (for our model it is uniformly apodized and normalized to $A_n = 1$).

β : Is the phase constant.

N : Is the Number of input **<u>or</u>** output electrodes.

X_n : Is the distance between the centers of the input/output *IDT*s and the zero *x*-axis.



(Where, $\mathbf{d_{1k}}$ is the distance between the first IDT input of the first rung and the *first IDT* of input rung k and $\mathbf{d'_{1k}}$ is the distance between the first *IDT* input of the first rung and the first *IDT* output of rung k)
Fig. 2: SAW filter of several input and output rungs.

Now, according to Eq. 1, the whole frequency response of the first input rung and the first output rung ([*F.R. Rung*] $_{11}$) will take the form:

$$[\text{F.R. Rung}]_{11} = [\text{F.R. Rung}]_{i1} \times [\text{F.R. Rung}]_{o1} = H_{i1}(f) \times H_{o1}(f) =$$

$$\sum_{n=1}^{N} (-1)^{n} \exp(-i\beta x_{n}) \sum_{m=1}^{M} (-1)^{m} \exp(-i\beta x_{m}) \tag{2}$$

By the same way, we can calculate the frequency response of every input rung with each output rung as shown below:

$[\text{F.R. Rung}]_{12} = H_{i1}(f) \times H_{o2}(f)$ *also* $[\text{F.R. Rung}]_{q1} = H_{iq}(f) \times H_{o1}(f)$
$[\text{F.R. Rung}]_{13} = H_{i1}(f) \times H_{o3}(f)$ $[\text{F.R. Rung}]_{q2} = H_{iq}(f) \times H_{o2}(f)$
$\vdots$ $\vdots$
$\vdots$ $\vdots$
$[\text{F.R. Rung}]_{1l} = H_{i1}(f) \times H_{ol}(f)$ $[\text{F.R. Rung}]_{ql} = H_{iq}(f) \times H_{ol}(f)$

and so on

Now, as the *SAW* filter is considered to be a linear one, so, it should obey the **super-position principle** (Oppenheim, 1997 and Sophocles, 1998), in turn, the summation of all the previous frequency responses will be equal to the whole frequency response of the linear *SAW* filter of Fig. 2.

$$\text{The SAW filter frequency response} = H(f) = [F.R.\ Rung]_{11} + [F.R.\ Rung]_{12} + \ldots + [F.R.\ Rung]_{1l} + [F.R.\ Rung]_{21} + [F.R.\ Rung]_{22} + \ldots + [F.R.\ Rung]_{2l} + \ldots + [F.R.\ Rung]_{q1} + [F.R.\ Rung]_{q2} + \ldots + [F.R.\ Rung] \quad (3)$$

The mathematical model of Eq. 3 can be written as the multiplication of the summation of the frequency responses of the input rungs with the summation of the frequency responses of the output rungs:

$$H(f) = \{([F.R.\ Rung]_{i1} + [F.R.\ Rung]_{i2} + \ldots + [F.R.\ Rung]\ iq) \times ([F.R.\ Rung]_{o1} + [F.R.\ Rung]_{o2} + \ldots + [F.R.\ Rung]_{ol})\} \quad (4)$$

Eq. 4 is the mathematical model that can be used to simulate the linear **matched** *SAW* filters of several combinations (one-to-one*, one-to-multi and multi-to-multi combinations).

* Note: One-to-one means: one input IDTs (or rung) with one output IDTs (or rung).

3. Simulating & testing the frequency response of One-to-Multi linear matched *SAW* filter design

Now, we will simulate and test of one-to-two linear **matched** *SAW* filter. We will use an input rung of nine electrodes of center frequency 161.92 MHz and an output of two rungs, the first has five electrodes of center frequency 161.92 MHz and the second has also five electrodes of center frequency of 194.3 MHz (see Fig. 3).
The frequency response of this linear **matched** *SAW* filter can be calculated from the derived mathematical model of Eq. 4 [see Eq. 5 and Eq. 6].

$$H(f) = ([F.R.\ Rung]_{i1}) \times ([F.R.\ Rung]_{o1} + [F.R.\ Rung]_{o2}) \quad (5)$$

$$H(f) = \sum_{n=1}^{9} (-1)^{n} \exp(-i\pi nf/161.92)[\sum_{m_1=1}^{5} (-1)^{m_1} \exp(i\pi m_1 f/161.92) \times \exp(i.\ \beta 412) + \sum_{m_2=2}^{5} (-1)^{m_2} \exp(im_2 f/194.3) \exp(i.\ \beta 585.5)] \quad (6)$$

Fig. 4 shows the simulation frequency response of one-to-two *SAW* filter design.

One-to-two design has been tested using the testing equipment HP8753D network analyzer (Issa, 2000). Fig. 5 shows the frequency response of the tested SAW filter.

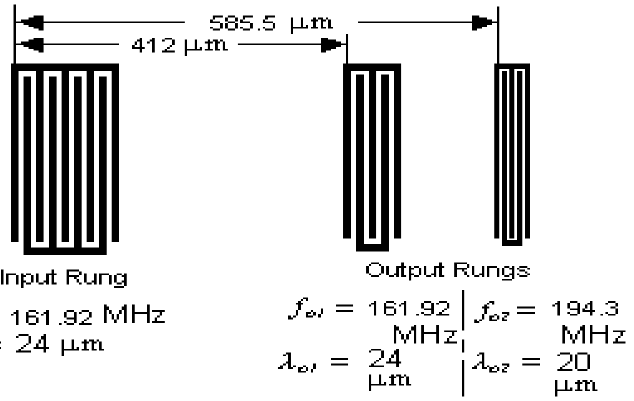


Fig. 3: The *SAW* filter design of one-to-two case

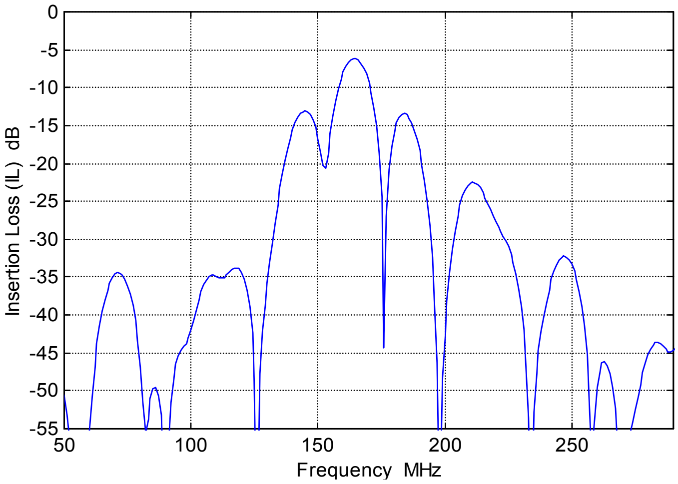


Fig. 4: The simulated frequency response of one-to-two linear matched *SAW* filter. The center frequency f_0 = 164 MHz, the insertion loss (IL) = -6. 23 dB and the 3-dB bandwidth = 10. 35 MHz.

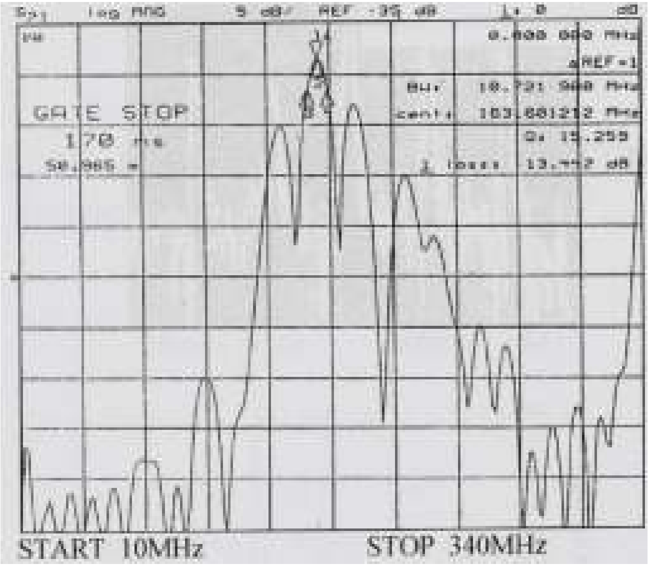


Fig. 5: The tested frequency response of one-to-two linear matched *SAW* filter. f_0 = 163. 6 MHz, 3-dB BW = 10. 32 and IL = -13. 44 dB.

4. Conclusion

In this work, we derived a mathematical model for linear SAW filters using the Delta function model. This model can be used to simulate the frequency response of all possible combinations of linear *SAW* filter designs. This model has shown similar center frequency and 3-dB BW for the simulated and the test results, but different in insertion loss, due to the time gated method (Campbell, 1998) that has been used in testing (this method will produce an extra insertion loss **that we can not reduce, control or evaluate**) (Issa, 2000). Finally, this model can be considered as a promising one for resolving the frequency response of the linear **matched** *SAW* filter designs.

5. References

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